

Exploring Dependency based Probabilistic Supply Chain Risk Measures for Prioritising Interdependent Risks and Strategies

Abroon Qazi^{a*}, John Quigley^a, Alex Dickson^b, Sule Onsel Ekici^c

abroon.qazi@strath.ac.uk; j.quigley@strath.ac.uk; alex.dickson@strath.ac.uk; sonsel@dogus.edu.tr

^aManagement Science Department, Strathclyde Business School, 199 Cathedral Street, Glasgow, Scotland, G4 0QU, United Kingdom

^bEconomics Department, Strathclyde Business School, 130 Rottenrow, Glasgow, Scotland, G4 0GE, United Kingdom

^cIndustrial Engineering Department, Dogus University, Acibadem Zeamet Sok, 34722 Istanbul, Turkey

*Corresponding author. Tel: +44 7435682387; fax: +44 141 552 6686

Abstract

In this paper, we introduce an integrated supply chain risk management process that is grounded in the theoretical framework of Bayesian Belief Networks capturing interdependency between risks and risk mitigation strategies, and integrating all stages of the risk management process. The proposed process is unique in four different ways: instead of mapping the supply network, it makes use of Failure Modes and Effects Analysis to model the risk network which is feasible for modelling global supply chains; it is driven by new dependency based risk measures that can effectively capture the network wide impact of risks for prioritisation; it utilises the concept of Shapley value from the field of cooperative game theory to determine a fair allocation of resources to the critical risks identified; and the process helps in prioritising potential risk mitigation strategies (both preventive and reactive) subject to budget and resource constraints. We demonstrate its application through a simulation study.

Keywords

Supply chain risk management; Bayesian Belief Networks; Failure Modes and Effects Analysis; risk measures; risk mitigation strategies

1. Introduction

Supply chains have become more complex due to the globalisation and outsourcing in manufacturing industries. Global sourcing and lean operations are the main drivers of supply chain disruptions (Son & Orchard, 2013). In addition to the network configuration based complexity, non-linear interactions between complex chains of risks categorised as ‘systemicity’ of risks (Ackermann, Howick, Quigley, Walls, & Houghton, 2014) make it a daunting task to understand and manage these dynamics. Supply chain risk management (SCRM) is an active area of research that deals with the overall management of risks ranging across the entire spectrum of the supply chain including external risk factors. Besides an increase in the frequency of disruptions, supply chains are more susceptible because of the increasing interdependency between supply chain actors and substantial impact of cascading events.

Supply chain risks can be viewed with respect to three broad perspectives: a ‘butterfly’ concept that segregates the causes, risk events and the ultimate impact; the categorisation of risks with respect to the resulting impact in terms of delays and disruptions; and the network based classification in terms of local-and-global causes and local-and-global effects (Sodhi & Tang, 2012). According to Manuj and Mentzer (2008b, p. 205), *“Global SCRM is the identification and evaluation of risks and consequent losses in the global supply chain, and implementation of appropriate strategies through a coordinated approach among supply chain members with the objective of reducing one or more of the following – losses, probability, speed of event, speed of losses, the time for detection of the events, frequency, or exposure – for supply chain outcomes that in turn lead to close matching of actual cost savings and profitability with those desired”*.

Risk management comprises different stages including risk identification, risk analysis, risk evaluation, risk treatment and risk monitoring (SA, 2009). A number of risk management frameworks have been proposed for managing supply chain risks (Chopra & Sodhi, 2004; Knemeyer, Zinn, & Eroglu, 2009; Manuj & Mentzer, 2008a; Sinha, Whitman, & Malzahn, 2004; Trkman & McCormack, 2009; Tummala & Schoenherr, 2011), however, there are two main limitations about these studies. The first and most significant limitation of these frameworks is their consideration of risks as independent factors. Classification of risks has been explored comprehensively resulting in identification of independent categories of risks for aiding the risk identification stage of SCRM process (Bogataj & Bogataj, 2007; Chopra & Sodhi, 2004; Jüttner, Peck, & Christopher, 2003; Kleindorfer & Saad, 2005; Manuj & Mentzer, 2008a; Oke & Gopalakrishnan, 2009; Tang & Tomlin, 2008). However, risk identification must involve different stakeholders and capture the interdependent interaction between risks across different domains of the stakeholders (Ackermann et al., 2014; Badurdeen et al., 2014). Studies focussing on interdependency between risks generally follow the process flow of the supply chain (Garvey, Carnovale, & Yenyurt, 2015; Leerojanaprapa, van der Meer, & Walls, 2013) which is not feasible when considering substantial supply chain networks.

The second limitation of the analysed frameworks relates to their main focus on the risk identification and risk analysis stages whereas risk *treatment* has not been explored in detail (Colicchia & Strozzi, 2012). Furthermore, limited studies have assessed risks within an interdependent setting and to the best of the authors' knowledge, only Garvey et al. (2015) have introduced probabilistic risk measures for interdependent supply chain risks and there is no study that explores interdependency between risks and risk mitigation strategies within a probabilistic network setting. These gaps that are found in the literature have led to the main research question that drives this research which is: How can we design a SCRM process capturing systemic interactions between risks and mitigation strategies across the integrated stages of risk identification, risk analysis, risk evaluation and risk treatment; and subsequently, how can the potential mitigation strategies be evaluated within the network of interdependent risks and strategies in relation to different resource and budget constraints? This research paper is a first step towards bridging this significant research gap. It attempts to propose risk measures and a process that can help researchers and practitioners appreciate the importance of capturing interdependency between risks and strategies across different stages of risk management process and develop better models for managing supply chain risks.

We achieve this by introducing a method of managing supply chain risks within a network setting of interacting risks, risk sources and mitigation strategies that is grounded in the theoretical framework of Bayesian Belief Networks (BBNs). For risk identification, we utilise the key feature of Failure Modes and Effects Analysis (FMEA) in identifying supply chain risks, associated sources and potential mitigation strategies. For risk assessment, we introduce dependency based probabilistic risk measures for

identifying the relative importance of each risk within the network of interacting risks. For risk treatment, we consider two scenarios: if the strategies and associated cost are not explicitly evaluated, we make use of Shapley value (Shapley, 1953) from the field of cooperative game theory in order to address the problem of allocating a fair amount of the budget to the critical risks identified through the measures; if the strategies with associated cost are already identified within the network, we focus on optimising strategies in relation to resource and budget constraints. We demonstrate the use of proposed process through a simulation study that is based on the case study of Tuncel and Alpan (2010).

The remainder of the paper is organised as follows: An overview of FMEA and BBNs is presented in section 2. Section 3 provides a brief review of the relevant literature. The proposed risk measures, propositions and the Shapley value based method are described in section 4. The proposed process is presented in section 5. The application of the proposed process is demonstrated through a simulation study in section 6. Finally, we conclude our paper with important findings and present future research themes in section 7.

2. FMEA and BBNs

2.1 FMEA

FMEA or Failure Modes, Effects and Criticality Analysis (FMECA) is a systematic approach of identifying different modes of failure and evaluating associated risks during the development stage of a product or service. It is known to have been implemented in 1963 for projects at NASA and later, the Ford Motor Company utilised the technique in 1977 (Gilchrist, 1993). The typical process involves: identification of failure modes, associated causes and resulting consequences; assigning the values of occurrence (O), severity (S) and detection (D) to each failure mode on an ordinal scale of 1-10 for each linguistic variable; calculating the Risk Priority Number (RPN) of each failure mode which is the product of three numbers identified previously; ranking the failure modes and planning actions on high ranking modes; and finally reviewing the effectiveness of implemented actions and revising the risk measures.

There are some major shortcomings of using RPN as a measure of prioritising risks (Gilchrist, 1993; Nepal & Yadav, 2015). The elicited value relative to each ordinal scale is quite subjective and furthermore, a risk having a high value of severity ($O=6, S=10, D=6$) might still score lower ($RPN=360$) in comparison with a risk ($O=6, S=8, D=8$) that might be less critical ($RPN=384$). Therefore, the calculation of RPN as a product of three numbers does not justify the rationale. In this study, we propose using the features of FMEA in identifying important risks and associated risk sources but instead of using the ordinal scales for occurrence and severity, we utilise the values of probability and losses resulting from realisation of risks. We also establish interdependency between identified risks and risk sources that helps in overcoming the notion of independent risks inherent in the conventional scheme of FMEA.

2.2 BBNs

BBNs are a framework for modelling uncertainty. BBNs have their background in statistics and artificial intelligence and were first introduced in the 1980s for dealing with uncertainty in knowledge-based systems (Sigurdsson, Walls, & Quigley, 2001). They have been successfully used in addressing problems related to a number of diverse specialties including reliability modelling, medical diagnosis, geographical information systems, and aviation safety management. For understanding the mechanics and modelling of BBNs, interested readers may consult Jensen and Nielsen (2007), and Kjaerulff and Anders (2008). We consider BBNs as the best choice of modelling technique in our situation as it facilitates capturing interdependency between risks and strategies.

There are a number of benefits associated with BBNs: firstly, these provide a graphical representation of the problem that can help stakeholders visualise the interaction between a number of variables; probabilistic reasoning is easily captured and propagated through powerful software and prior beliefs about the uncertain variables can be easily updated after providing evidence against separate sources in the network; uncertainty in reasoning is taken into account and the (in)dependence between variables can be recognised; and one can model BBNs even when there is limited empirical data. However, there are some shortcomings of the method as well: elicitation of expert judgment in both developing and populating the network is challenging when data is not readily available; available software have limited capability in dealing with continuous variables as the variables have to be discretised which can lead to a limited ability to capture the original distribution of the variable; and the “acyclic graph” requirement, which is needed to carry out probability calculus, is another limitation that results in feedback effects not being included in the network (Jensen & Nielsen, 2007).

3. Literature Review: Models for Managing Interdependent Supply Chain Risks

As the research question investigates development of a SCRM process considering interdependency between supply chain risks and mitigation strategies, the focus will be limited to the literature dealing with interdependent risks. For a comprehensive overview of quantitative models in SCRM, interested readers may consult the literature review conducted by Fahimnia, Tang, Davarzani, and Sarkis (2015). A number of models have been proposed for identifying and assessing supply chain risks, however, limited studies have considered interdependency between risks. Cause-effect diagram (Lin & Zhou, 2011) and social network theory (Kim, Choi, Yan, & Dooley, 2011) have been used for mapping causal interaction between supply chain risks. Interpretive structural modelling has been used for modelling interdependency between risks (Pfohl, Gallus, & Thomas, 2011) and identifying the interdependent enablers of risk mitigation (Faisal, Banwet, & Shankar, 2006) which helps in not only mapping the relationship between variables but also in developing a hierarchy of the network. The main problem with these techniques is the inability of modelling the strength of relationship between interconnected risks.

FMEA has been used for identifying and assessing supply chain risks (Nepal & Yadav, 2015; Tuncel & Alpan, 2010). The major shortcoming of these studies is the use of RPN for ranking risks (Gilchrist, 1993) and failure to capture the network wide propagation of risks. Supplier selection/assessment has remained one of the active areas of research and a number of methods including Analytical Hierarchy Process (AHP) (Chen & Wu, 2013) and BBNs (Dogan & Aydin, 2011) have been developed to assess supplier related risks. The main limitation of these studies is their focus on addressing a specific problem without considering the holistic interaction of risks across the supply network (Garvey et al., 2015).

The likelihood of the occurrence of an (undesirable) event, and the negative implications of the event are two common measures of risk (Bogataj & Bogataj, 2007). Risk mitigation strategies are implemented in order to reduce the likelihood of occurrence and/or negative impact of risks (Tang & Tomlin, 2008). Robust strategies must be developed in order to help firms reduce cost and/or improve customer satisfaction under normal conditions and enable firms to sustain operations during and after the disruption. A number of studies have proposed selecting strategies specific to the supply chain configuration and risks (Christopher & Lee, 2004; Christopher, Mena, Khan, & Yurt, 2011; Son & Orchard, 2013; Speier, Whipple, Closs, & Voss, 2011; Zsidisin, Ellram, Carter, & Cavinato, 2004). Few studies (Aqlan & Lam, 2015; Micheli, Mogre, & Perego, 2014) have considered the optimisation problem of selecting cost-effective risk mitigation strategies, however, no study has ever considered the problem of evaluating optimal combinations of risk mitigation strategies within a probabilistic network setting of interacting risks and strategies.

BBNs have been extensively applied to the field of risk management (Ashrafi, Davoudpour, & Khodakarami, 2015; Norrington, Quigley, Russell, & Van der Meer, 2008; Wu, Yang, Chang, Château, & Chang, 2015) mainly because BBNs offer a unique feature of modelling risks combining both the statistical data and subjective judgment in case of non-availability of data (Dogan & Aydin, 2011). However, their application to the field of SCRM in modelling holistic interaction between risks has recently gained the interest of researchers (Garvey et al., 2015; Leerojanaprapa et al., 2013). Badurdeen et al. (2014) introduced a supply chain risk taxonomy and a risk network map capturing interdependency between risks. Their model presents an effective tool to capture the interaction of risk factors and helps in identifying critical suppliers.

In a recent study conducted by Garvey et al. (2015), supply chain process and risks corresponding to various segments of the supply network are combined together and modelled as a BBN. They also introduce new risk measures for identification of important elements within the supply network. Their proposed modelling framework differs from the existing BBN based studies in SCRM (Badurdeen et al., 2014; Dogan & Aydin, 2011; Lockamy, 2014) in terms of exploring the propagation impact of risks across the network of interconnected risks and supply network elements, but their proposed risk measures only consider the impact of risks on the descendant nodes and ignore capturing the diagnostic effect.

They also incorporate the loss values within their modelling framework thereby overcoming the major limitation of earlier studies in terms of focusing on only the probabilistic interdependency between risks. However, the proposed framework does not focus on modelling and evaluating risk mitigation strategies (risk treatment). Furthermore, it might not be feasible to adopt the method for mapping a huge network as the method necessitates following the process flow of the supply chain.

Heckmann, Comes, and Nickel (2015) conducted a critical review of quantitative approaches for managing supply chain risks focussing on the definitions, measures and modelling of risk. According to them: *'Standard deviation, mean-variance approaches, value-at-risk, conditional-value-at-risk or premiums are risk measures that aim at describing the interaction of uncertainty and the extent of its related harm or benefit. Owing to the lack of quantitative measures that capture the more complex realities of supply chains, these measures – developed in finance and insurance contexts – are applied for supply chain risk, too'* (Heckmann et al., 2015, p. 127). However, a closer look at the cited references in their study reveals that the measures are not developed for interdependent risks and that is why the risk measures introduced by Garvey et al. (2015) are deemed as state-of-the-art in terms of capturing the interdependency between risks and *'measuring monetary losses within supply chain management'* (Heckmann et al., 2015, p. 128). However, Garvey et al. (2015) rightly identify the limitation of their proposed measures as these only capture propagation of losses across the pure descendants of risks (causal effect) rather than evaluating the network wide propagation of losses (causal and diagnostic effects).

Although our study can be considered as an extension to the study conducted by Garvey et al. (2015) in terms of exploring BBNs as a framework for managing supply chain risks, there are some major differences. Our contribution to the literature in SCRM is multi-faceted: we introduce a comprehensive integrated process of SCRM grounded in the theoretical framework of BBNs and to the best of the authors' knowledge, a probabilistic graph integrating all stages of the risk management process and capturing interdependency between risks and strategies has never been explored; we propose dependency based probabilistic risk measures capturing network wide impact of risks that help in prioritising risks both in the risk assessment and risk monitoring stages; we utilise the concept of Shapley value to determine fair allocation of resources to the critical risks identified; and we establish a method of prioritising risk mitigation strategies within a probabilistic network setting.

4. Metrics to Support Resource Allocation and Their Characteristics

In this section we start in 4.1 with a simple illustrative example to motivate the measures we propose for assessing risk on the network. In section 4.2 we explore characteristics of these measures and reflect on their applicability in defining appropriate risk mitigation strategies for a network. In section 4.3 we explore characteristics of an optimal portfolio of risks subject to a budget constraint. Lastly in section 4.4

we consider the use of Shapley value on the network to identify fair budget allocations prior to developing risk mitigation strategies.

4.1 Motivating Example

Consider a supply network with three identified risks and an associated BBN illustrated in Fig. 1. Risk 1 (R1) and Risk 3 (R3) have no parent nodes with a probability of being realised of 0.5 and 0.2, respectively. Risk 2 (R2) is dependent on both R1 and R3 with Conditional Probability Table (CPT) provided in Table 1, and a marginal probability of being realised of 0.429. Associated with R1, R2 and R3 are a Loss 1, 2 and 3 of 100, 1000 and 500 respectively if the risk is realised. This produces a correlation between Loss 1 and Loss 2 of 0.34 and Loss 2 and Loss 3 of 0.53.

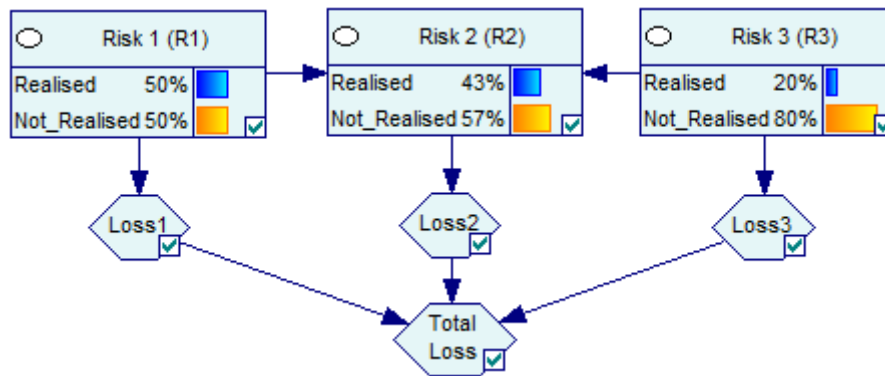


Fig. 1. A Bayesian Belief Network illustrating three risks each with an associated loss node and a total loss node (GeNIe, 2015)

Table 1. Conditional probability table of Risk 2

		State of Risk 1			
		Not Realised (0.5)		Realised (0.5)	
		State of Risk 3		State of Risk 3	
		Not Realised (0.8)	Realised (0.2)	Not Realised (0.8)	Realised (0.2)
State of Risk 2	Realised	0.1	0.9	0.5	0.99
	Not Realised	0.9	0.1	0.5	0.01

The expected direct loss from R1, R2 and R3 is 50, 429 and 100 respectively with an Expected Total Loss of 579 and a standard deviation of 638. We shall refer to the Expected Total Loss as the Risk Network Expected Loss (*RNEL*) to reflect that the loss represents a total loss across the network of risks after accounting for the propagation of risks through the network. Illustrated in Fig. 2 is the probability distribution for the realised Total Loss, so while the mean of this distribution is 579, the probability of realising a total loss in excess of this is 0.43, of realising a loss of at least twice the mean is 0.389 and there is a probability of 0.099 that the total loss will be 1600, almost three times the mean.

Decision makers may have resources available to wholly or partially mitigate a risk, in which case assessing the impact a risk has on Network Loss becomes important. This is a challenging exercise in the presence of dependency or correlation between the direct losses, as once realised a risk can propagate consequences, increasing the likelihood of realising other risks. Fig. 3 illustrates three probability functions representing the distribution with R1, R2 or R3 entirely mitigated (i.e. the probability of it

being realised is set to zero). Key summary statistics of these distributions are provided in Table 2. The distributions are quite different which is not reflected in measures such as *RNEL*. No distribution stochastically dominates, so choosing the most important risk to manage will depend on the preferences of the decision maker: how mitigating each risk is valued by the decision maker depends on their assessment of the value of the change in the probability distribution that materialises.

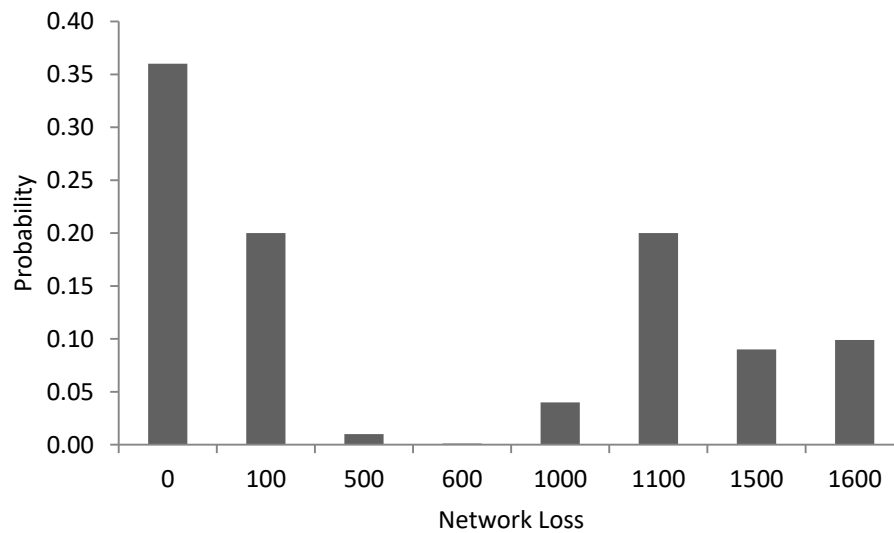


Fig. 2. Probability distribution of Network Loss

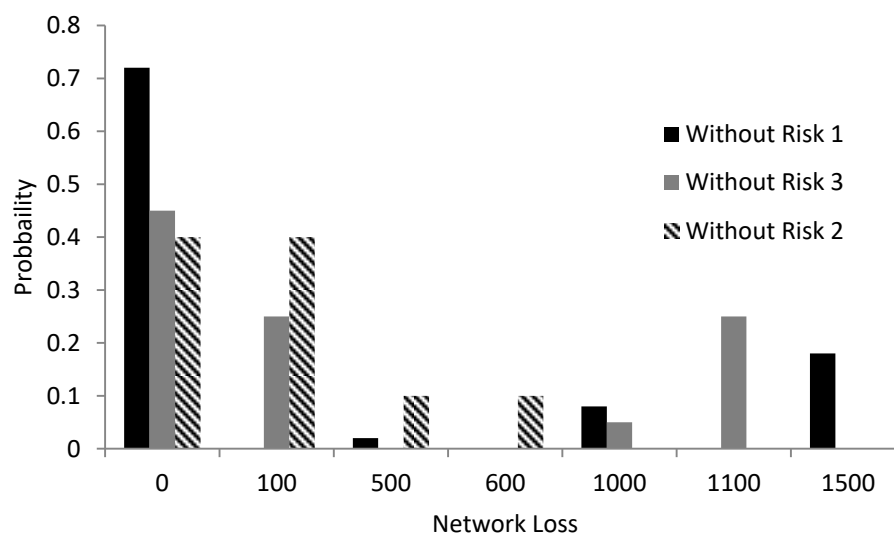


Fig. 3. Probability distribution of Network Loss assuming Risk 1, Risk 2 or Risk 3 is removed showing a variety of shapes

Table 2. Summary Statistics from the distribution of Network Loss assuming Risk 1, Risk 2 or Risk 3 is removed

Risk Removed	RNEL	Standard Deviation	Best Case (probability)	Worst Case (probability)
None	579	638	0 (0.36)	1600 (0.099)
Risk 1	360	600	0 (0.72)	1500 (0.18)
Risk 2	150	206	0 (0.45)	600 (0.10)
Risk 3	350	482	0 (0.40)	1100 (0.25)

Consider the conditional distributions of Network Loss given a risk has been realised (i.e. its probability is set to one). For the simple illustrative example in this section the three conditional distributions are provided in Fig. 4. Note that, due to the direction of the causal relationship between R1 and R3, and R2, if R2 is realised the probability of R1 and R3 are not updated as there is no change in epistemic uncertainty. It is clear from this illustration that the influence of each risk on possible network losses is very different: for example, the expected loss if R3 is realised is much higher than if R2 is realised. Whilst *RNEL* gives an *ex ante* measure of losses that are at stake on the network, it does not allow any inference about the importance of individual risks. In order to do this we propose the Risk Network Expected Loss Propagation Measure for Risk *i* (*RNELPM_i*), which measures the probability-weighted *RNEL* if risk *i* is realised. Table 3 provides a summary of the distributions illustrated in Fig. 4 along with the *RNELPM* for each risk.

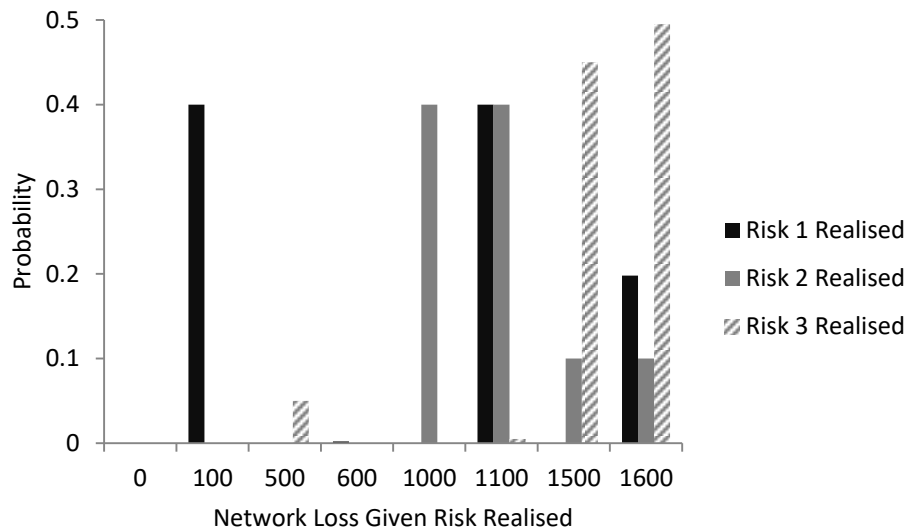


Fig. 4. Probability distribution of Network Loss given Risk 1, Risk 2 or Risk 3 is realised

Table 3. Summary Measures of Network Loss given risks realised

Risk Realised	RNEL given risk realised	Probability of realising risk	RNELPM	Difference between RNEL given risk realised and RNEL	UTC
Risk 1	798	0.50	399	219	110
Risk 2	1150	0.43	495	571	246
Risk 3	1498	0.20	300	919	184

The idea of proposing the *RNELPM* is to allow decision makers to prioritise the reduction of risk on the network if resources are available to do so by identifying those risks that have the greatest effect on the network expected loss given the propagation of risks through the network, also accounting for the likelihood of them occurring. This takes the mean of the distribution for each risk in Fig. 4 and weights it by the probability of the risk occurring. If decision makers are risk-neutral their assessment of the distribution is correctly summarised by the average. If a decision maker has non-neutral risk preferences, it would not be appropriate to use the expected loss to summarise the distribution, but to consider the expected utility of the loss: a probability-weighted average of the utility of the losses that

might be realised if a risk is realised, which should then itself be weighted by the probability of the loss occurring. Eliciting a decision maker's utility function is known to be challenging (Ruan, Yin, & Frangopol, 2015) leading to difficulties in operationalising such a measure and so, whilst it may be interesting to pursue this in future work, we turn our attention to a different line of inquiry.

There is lots of evidence to suggest that human decision makers evaluate the outcomes from the choices they make by comparing those outcomes relative to a reference outcome, and exhibiting 'loss aversion'. This is the characteristic that if an outcome is a certain amount worse than the reference outcome then this gives a greater reduction in the attractiveness of the outcome than the increase in attractiveness if an outcome is the same amount better than the reference outcome. The idea of loss aversion was made famous by Kahneman and Tversky (1979) in their 'Prospect theory', which has been developed in particular by Kőszegi and Rabin (2006), who carefully consider the use of expectations as reference points, and applied extensively to many interesting scenarios.

In the context of a network of interdependent supply chain risks, it is far from inconceivable that a supply chain manager may have in mind an expected loss for the standard configuration of the network, and in evaluating the importance of particular risks may place more emphasis on those risks that increase the network expected loss above the expected loss for the standard configuration, which is taken as the 'reference loss'. While managers may have differing degrees of loss aversion, a straightforward way to capture this is to evaluate the impact of a risk being realised by focussing only on where realisation of that risk leads to losses that exceed the reference loss, and ignore instances where the loss falls below the reference loss. In the case of our simple example, we can easily evaluate the Expected Loss in Excess of the Mean (*ELEM*), i.e. $E[\max(\text{Network Loss} - RNEL, 0)]$, using the distribution in Fig. 2 to obtain 305. However, for more complex networks this measure becomes computationally burdensome, and as an approximation we consider that the decision maker is concerned with the maximum of the difference between the *RNEL* if the risk is realised and the reference loss, and zero, which we define as the 'Upper Tail Contribution' (*UTC*) of a risk. By isolating expected losses in excess of the reference loss, this measure provides an alternative assessment of risk that captures the importance of reference dependence and loss aversion in the evaluation of risks. The calculations for these measures are given in Table 3.

While *RNELPM* and *UTC* are similar in emphasis of purpose, upon comparing the summary measures in Table 3 we see that *RNELPM* provides a different rank to the risks than *UTC*. Overall, R2 comes out as being high in importance, but using *RNEL* R1 and R3 are marginally different, *RNELPM* has R1 being more important than R3, and the opposite is true with *UTC*. In the following section we will formally define these measures and explore their characteristics.

4.2 The Risk Measures and Their Properties

In this section we will be concerned with elucidating the properties of the three measures introduced in section 4.1. These are the $RNEL$ (1), which is the expected total loss on the network, the UTC_i (2), which is the expected increase in $RNEL$ from realising risk i and $RNELPM_i$ (3), which is the expected network loss from realising risk i . Our intention is to investigate the applicability of these measures within an optimisation algorithm to determine the cost optimal level to target the probability of each risk. All proofs are contained in the Supplementary Material (see Appendix A).

$$RNEL = E[NL] \quad (1)$$

$$UTC_i = E_{R_i}[\max(E[NL|R_i = 1] - E[NL], 0)] \quad (2)$$

$$RNELPM_i = E[NL|R_i = 1]P(R_i = 1) \quad (3)$$

We start with making observation 1, where using $RNEL$ as a reference point, realising a risk will occur increases the updated $RNEL$ and realising a risk will not occur decreases the updated $RNEL$ for all risks.

Observation 1:

$$E[NL|R_i = 0] \leq RNEL \leq E[NL|R_i = 1]$$

Observation 2 establishes that UTC_i will never exceed the Expected Loss in Excess of the Mean ($ELEM$).

Observation 2:

$$ELEM \geq UTC_i$$

Observation 3 establishes the relationship between the three measures for a specific risk on the network, and the expected loss on the network. As such a risk with a higher probability of being realised as a loss has a greater difference between $RNELPM_i$ and UTC_i .

Observation 3:

$$\frac{RNELPM_i - UTC_i}{P(R_i=1)} = RNEL, \forall i$$

This leads to the first proposition concerning UTC_i , which shows the relationship between UTC_i and $ELEM$ with respect to the probability of experiencing a network loss below $RNEL$. The second proposition is motivated by focussing on the network losses that are in excess of a reference point, namely the $RNEL$. We can define the Lower Tail Gain for risk i , to be the expected gain from realising network losses below the reference point. The equivalence of these measures is expressed in Proposition 2.

Proposition 1: As the conditional probability of realising an aggregate network loss below the $RNEL$ given risk i has been realised, i.e. $P(\text{Network Loss} < RNEL | R_i = 1)$, decreases UTC_i approaches $ELEM$, i.e. $\lim_{P(\text{Network Loss} < RNEL | R_i = 1) \rightarrow 0} UTC_i = ELEM$.

Proposition 2: *The Upper Tail Contribution for risk i (UTC_i) equals the Lower Tail Gain for risk i (LTG_i), i.e. $UTC_i = E_{R_i}[\max(E[NL] - E[NL|R_i = 0], 0)]$.*

The third proposition explicates the relationship between UTC_i and the variance of its associated risk denoted by σ_i^2 .

Proposition 3: *UTC_i is proportional to the variance of the indicator variable for the risk, specifically*

$$UTC_i = \sigma_i^2 [E[NL|R_i = 1] - E[NL|R_i = 0]].$$

4.3 Optimal Control of Risk

We now consider that a supply chain manager has been allocated a budget that can be used to reduce risk on the network, and consider the optimal way in which to reduce risk. We suppose that risks are controllable, in the sense that the manager can undertake costly actions to reduce the probability that a risk is realised, or indeed perhaps release some cost by allowing the probability of a risk being realised to increase. For ease of notation, let $P_i = P(R_i = 1)$ and define $C_i(P_i)$ as the cost of achieving P_i . Since we are considering optimising from the standard configuration, we define the cost in the standard configuration as zero for each risk, and further suppose that for each i $C_i(\cdot)$ is continuously differentiable as many times as required and strictly decreasing in its argument (i.e. $C'_i(\cdot) < 0$), meaning it is costly to reduce the probability of a risk being realised. We further suppose that the cost function is convex (i.e. $C''_i(\cdot) > 0$), implying that the incremental cost of further reductions in the probability of a risk being realised is higher the smaller the probability is. A risk mitigation problem will involve minimising a risk measure subject to the total cost of risk mitigation not exceeding the manager's budget constraint, that we denote c_0 .

While UTC is a risk measure that captures reference dependence and loss aversion in decision making, an unfortunate consequence of Proposition 2 is that it does not make an effective decision making tool; as we show in the following corollary it can lead to very poor decisions being made.

Corollary 1: *If $C_i(P_i)$ is decreasing in P_i , optimising a portfolio of risks with the objective of $\min \sum_i UTC_i$ with respect to P_i will lead to maximising $E[NL]$.*

Instead, we turn to consider $RNELPM_i$ as a risk measure to guide the management of risks on an interdependent supply network. In Proposition 4 we characterise the relationship between the optimal level to target probabilities of risks being realised in relation to $E[NL|R_i = 1]$.

Proposition 4: *Optimising a portfolio of risks with the objective of $\min \sum_i RNELPM_i$ with respect to P_i subject to a budget constraint results in an optimal P_i such that marginal cost at P_i is proportional to $E[NL|R_i = 1]$ for all i .*

The optimal risk mitigation strategy calls for the marginal benefit of incrementally reducing a risk being realised to be weighed up against the marginal cost of further reductions in the probability, accounting

for the fact that the budget is constrained. When optimising from a standard configuration with a fresh budget a manager may optimally reduce or increase certain risks. The solution to the optimisation problem allows us to consider optimal risk realisation probabilities as a function of the budget, and inspection of this relationship reveals that under our assumptions a relaxation of the budget constraint will result in the probability of all risks materialising being reduced. As such, while the further reduction of certain risks might be favoured over others, no risk will see an increase in the probability of it materialising. This is formalised in Proposition 5. Lemma 1, which is used in the proof of this proposition, characterises the relationship between the marginal benefit of increasing the budget, denoted by λ^* , and the budget, denoted by c_0 , with the curvature of the cost function.

Lemma 1: Assuming $C'_i(P_i) < 0$ and $\text{sign}(C''_i(P_i)) = \text{sign}(C''_j(P_j)) \forall i, j$ then

$$\text{sign}\left(\frac{d\lambda^*}{dc_0}\right) = -\text{sign}(C''_i(P_i))$$

Proposition 5: If the cost function of changing the probability of each risk being realised is convex, then increasing the budget, c_0 , for risk mitigation will not result in decreased optimal probability for any risk, $P_i^*(c_0)$. Specifically,

$$\frac{dP_i^*(c_0)}{dc_0} = \frac{1}{C'_i + \frac{C''_i}{C'_i} \sum_{j \neq i} \frac{C_j'^2}{C_j''}} < 0, \forall i$$

In section 6 we will investigate an illustrative case on a much larger network than the one explored in section 4.1. We will determine the optimal risk mitigation strategy, which is in fact, identifying the optimal target levels to reduce the risks to subject to budget constraints. Such decision making requires that we can express costs as a function of these probabilities. Such a model requires the risk mitigation strategies to be quite advanced in their planning. Prior to this, we do require budgets to be allocated to risks about which we can plan such activities. Tackling this decision problem is addressed in section 4.4.

4.4 Shapley Value

We make use of Shapley value to determine the relative contribution of controlling each risk to the overall reduction in the risk network expected loss. The Shapley value, having its roots in cooperative game theory, has been applied to various problems including environmental pollution cost allocation, production decisions, transportation, allocation of electricity transmission costs and insurance pricing (Quigley & Walls, 2007). It has also been applied for trading reliability targets between supply chain partners in an aerospace industry (Quigley & Walls, 2007). Shapley derived a formula for evaluating the contribution of a player to the value of a cartel in a cooperative game (Shapley, 1953).

We adapt the cooperative game theory setting to our problem of allocating resources to critical risks. Individual risks (and associated controls) are the players, cartel is represented by the coalition of risk

controls applied to the specific risks and value corresponds to the relevant benefit in reducing the risk network expected loss. Any risk which is not the member of a network of controlled risks (coalition) is considered to be in its current (uncontrolled) state. As the formula for evaluating Shapley value is based on three axioms, we adapt these to the setting of SCRM as follows:

1. The benefit (reduction in risk network expected loss) attributed to the contribution of a risk control depends upon whether the risk control is implemented or not, and does not depend on the order in which the control was included in the set of risk controls.
2. The sum of the benefits attributed to the individual risk controls should equal the benefits made within the set of risk controls, with controls making no contribution to the set of controls being assigned zero value.
3. There is no expected loss or gain in delaying the implementation of a risk control at any given decision point.

It is assumed that the number of risk controls to be considered is specified *a priori* and is denoted by $|N|$. Let Z represent the set of risk controls that have already been implemented prior to implementing the risk control i and $|Z|$ is the corresponding number of risk controls. The benefit arising from implementing the risk control i to a network of size $|N|$ is given by the Shapley value (Shapley, 1953):

$$\Phi_i = \sum_{Z \in N-i} \frac{|Z|! (|N| - |Z| - 1)!}{|N|!} [v(Z \cup \{i\}) - v(Z)] \quad (4)$$

Where $v(Z \cup \{i\})$ represents the benefit (reduction in risk network expected loss) of implementing risk controls Z and control i , $v(Z)$ is the benefit of implementing controls Z ; $|Z|$ and $|N|$ indicate the number of elements in the sets Z and N respectively. Shapley value is a weighted average of the marginal contribution risk control i makes to a coalition, averaged over all possible permutations of entry to the coalition. The weights represent the probability of formation of a coalition of size Z prior to the implementation of risk control i . The calculation of Shapley value for the risk network (see Fig. 1) is shown in Table 4.

Table 4. Relative benefit of controlling each risk toward reduction in risk network expected loss

Control Risks $Z \cup \{i\}$	Expected Loss	Benefit of Control	Marginal Contribution $[v(Z \cup \{i\}) - v(Z)]$			Weight $\frac{ Z ! (N - Z - 1)!}{ N !}$
			$i = R1$	$i = R2$	$i = R3$	
0	579	0				
R1	360	219	219			1/3
R2	150	429		429		1/3
R3	350	229			229	1/3
R1, R2	100	479	50	260		1/6
R1, R3	100	479	250		260	1/6
R2, R3	50	529		300	100	1/6
R1, R2, R3	0	579	50	100	100	1/3
<i>Shapley value Φ_i</i>			139.67	269.66	169.67	
<i>Relative importance (%)</i>			24.1	46.6	29.3	

It is clear from the calculations that controlling R2 will be most beneficial to the network whereas controlling R1 or R3 is relatively less important. These values help in evaluating fair allocation of budget to the risks. The method captures all possible combinations of risk interactions. Shapley value provides a fair allocation of resources for risk mitigation as a starting point. Consider a situation where we have two risks (R1, R2) each with probability 1 of causing loss and the total loss is 1 unit regardless of the cause, i.e. only one or both. The Shapley value would be 0.5 for each risk but the risk is not reduced by 50% through eliminating R1 or R2. Therefore, if we have a budget B, then Shapley value suggests an initial proposal would be to allocate B/2 to each risk. However, we might be able to mitigate R1 for B/4 and spend 3B/4 on R2 and this could be an optimal allocation of the budget. So the optimisation aspect plays a different role and requires plans to be costed.

4.5 Summary

In this section we have proposed and illustrated new measures from assessing the contribution of risks to the aggregate loss across the network subject to dependency. The measures we considered in sections 4.1 and 4.2 are concerned with explicating the relationship between excess of losses on a network above a point of reference such as *RNEL* and the probabilities associated with these risks. We followed this by considering how to initially identify a fair allocation of resources to mitigate risks before mitigation strategies have been developed through Shapley value. In section 5 we will formally present our integrated process for supply risk management and in section 6 illustrate its application.

5. Proposed Risk Management Process

The proposed process comprises three main stages of problem structuring, instantiation and inference as shown in Fig. 5. The model can be developed through conducting interviews and focus group sessions with the experts. Although we make use of FMEA, the criticism related to the subjective nature of RPN (Liu, Liu, & Liu, 2013) is not relevant to our method because the FMEA is just utilised for identifying risks, sources and mitigation strategies. As the complete information or data concerning risks is generally not available, there is always a need to involve experts in modelling both the qualitative and quantitative parts of the model which makes the process quite subjective. However, any method will have to rely on expert judgment in case of non-availability of data and as our method is grounded within the framework of BBNs, well-established procedures and protocol can be adopted in order to develop and validate the model (Nadkarni & Shenoy, 2004; Pitchforth & Mengersen, 2013).

For better understanding, a block diagram is presented as Fig. 6 which manifests the contribution of this study to the established risk management process (SA, 2009). Although we demonstrate the application of the model for a one-time decision problem of prioritising risks and mitigation strategies (at time: $T = t_0$), it can easily be extended to monitor and re-evaluate risks and strategies periodically. For a

detailed discussion on each stage of the risk management process, interested readers may consult SA (2009).

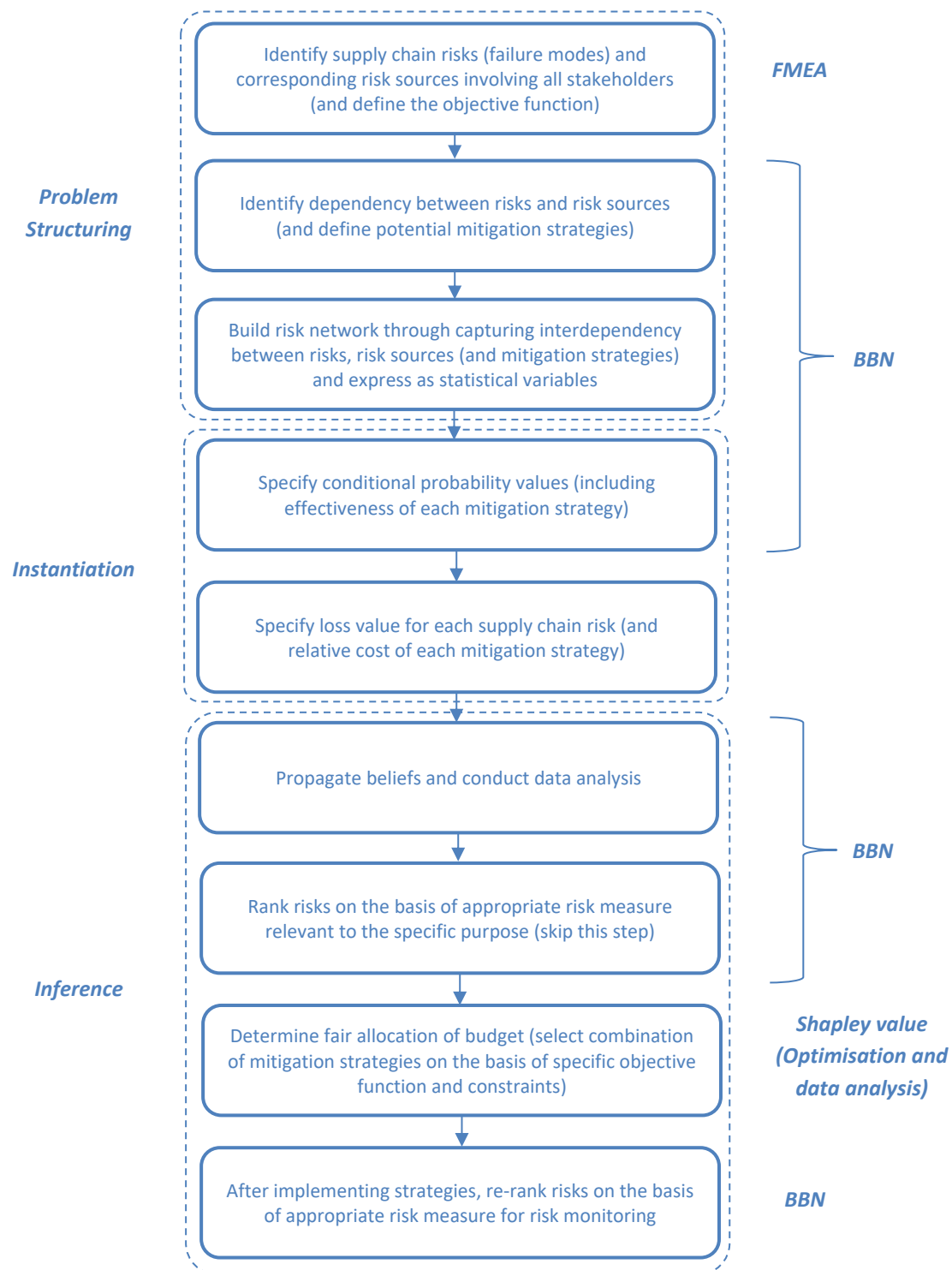


Fig. 5. Modelling flowchart of the proposed process (steps in brackets are applicable to scenario 2 only where mitigation strategies and associated cost are already established in the problem structuring)

The proposed process fits well with two distinct scenarios: in scenario 1, risk mitigation strategies and associated cost are not pre-defined; while in scenario 2, the strategies and associated cost are already established within the problem structuring stage. In both scenarios, our proposed risk measures help in prioritising critical risks for the risk monitoring stage. If the potential risk mitigation strategies are

already identified within the network setting with associated cost and efficacy in mitigating risks, we do not need to assess risks before implementing strategies as each combination of strategies would have a unique impact on the risk network and therefore, it makes sense to re-evaluate risks after selecting optimal strategies. Once the strategies are not already defined, we need to identify critical risks using an appropriate risk measure and subsequently determine fair allocation of resources to mitigate the critical risks using Shapley value. The detailed flow charts for the two scenarios are presented in the Supplementary Material (see Appendix B).

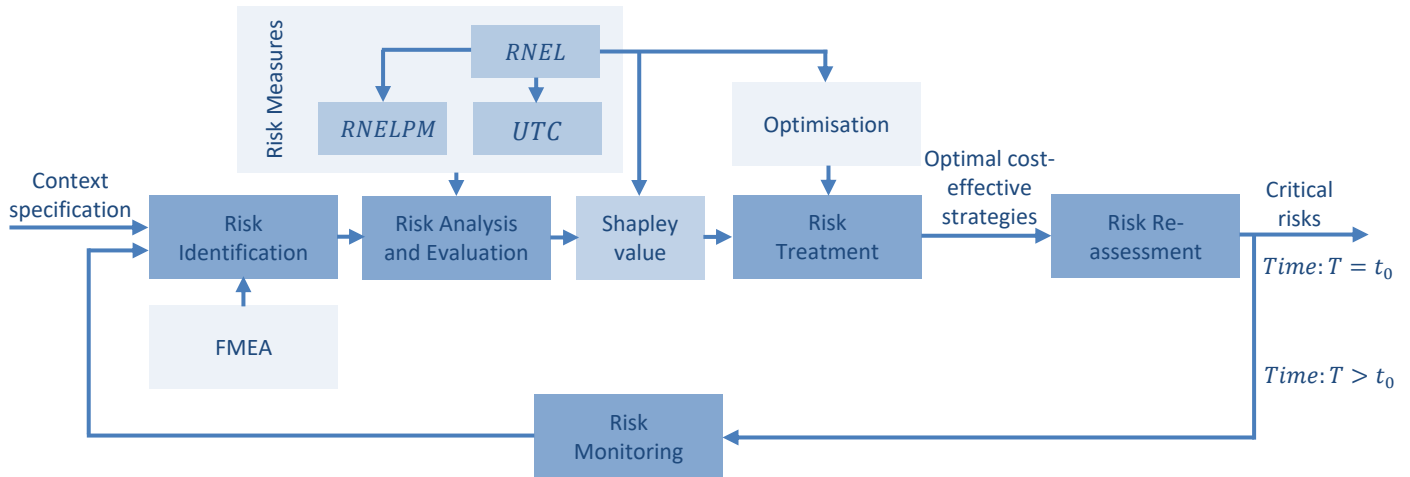


Fig. 6. Block diagram representing the integration of proposed methodology in the risk management process

5.1 Stages of the Process

5.1.1 Problem Structuring

Firstly, supply chain risks (failure modes) and associated risk sources are identified using the FMEA. In the case of scenario 2, the objective function is also defined taking into account the budget and/or resource constraints. The second step involves identifying interdependency between common risk sources and risks using the technique of cognitive mapping besides selecting potential mitigation strategies in case of scenario 2. Finally, the network structure is developed through connecting the arcs across related risk sources, risks and mitigation strategies (if applicable) and all nodes are expressed as statistical variables. The problem owner needs to ensure that the model is developed to represent the actual interdependency between risks. The model builder can assist in structuring the model keeping in view the mechanics of a BBN as the problem owner might not understand the importance of establishing correct relationships between causes and effects.

5.1.2 Instantiation

This stage involves evaluation of (conditional) probabilities (including effectiveness of mitigation strategies in case of scenario 2) either through elicitation from the experts or extraction from available data. Probability elicitation is the most difficult task of the modelling process as experts find it

challenging to describe conditional probabilities. Loss values are also elicited for all the risks and the cost of each mitigation strategy is ascertained through expert judgment in case of scenario 2.

5.1.3 Inference

In the case of scenario 1, key risks are identified through evaluating specific risk measures suitable for the purpose: *RNELPM* is suitable for capturing a risk-neutral appetite; whereas *UTC* is suitable for modelling risk-averse attitude where extreme losses are of greater concern. Once critical risks are identified, Shapley value is used for assigning resources to mitigate risks as well as comparing if the risk mitigation strategies are well priced. In the case of scenario 2, beliefs are updated and propagated across the interconnected risks, risk sources and mitigation strategies. For each possible combination of strategies, the network wide parameter *RNEL* is evaluated and cost and benefit analysis of various combinations of mitigation strategies is conducted. Depending on the objective function and constraints, appropriate strategies are selected. In both scenarios, once mitigation strategies have been evaluated (risk treatment), it becomes more important to re-assess the risks after implementation of strategies as the strength of interdependency between risks is reduced and the new network yields relatively independent risks. Therefore, an appropriate risk measure is used to prioritise critical risks for the monitoring stage and developing contingency plans.

5.2 Optimisation of a Portfolio of Risk Mitigation Strategies

We also investigate an important aspect of selecting optimal risk mitigation strategies within a network of interacting risk sources, risks and mitigation strategies subject to resource and budget constraints. Although we just make use of *RNEL* within the objective function that reflects the risk attitude of a risk-neutral decision-maker, the function can be tailored for capturing other risk attitudes with the addition of constraints like mitigating critical risks identified through the proposed risk measures. The following two problems relate to different constraints: the first considers optimising a portfolio of strategies subject to resource constraint; whereas the second relates to the optimisation problem subject to a budget constraint.

Problem No. 1

Given different options of implementing preventive and reactive strategies across a network of interconnected risk sources, risks and strategies, what is the optimal combination of these strategies yielding maximum (minimum) value of an objective function subject to resource constraint?

Objective function. In this study, we consider the following objective functions:

$$\min_{\gamma_{xs} \in \gamma_{xs}} RNEL_{\gamma_{xs}} \quad (5)$$

$$s. t. 0 < n \leq N$$

$$\max_{\gamma_{xs} \in \gamma_{xs}} RNEL_{SC} - RNEL_{\gamma_{xs}} - C_{\gamma_{xs}} \quad (6)$$

$$s. t. 0 < n \leq N$$

where N is the total number of potential mitigation strategies,

$RNEL_{SC}$ is the risk network expected loss under standard configuration of risk network (with no potential strategy implemented),

γ_{X_S} is a set of all possible orderings of different states of N mitigation strategies,

$C_{\gamma_{X_S}}$ is the cost of implementing γ_{X_S} combination of mitigation strategies,

n is the number of strategies being considered for implementation.

Problem No. 2

Given different options of implementing preventive and reactive strategies across a network of interconnected risk sources, risks and mitigation strategies, what is the optimal combination of these strategies yielding minimum value of an objective function subject to budget constraint?

Objective function. In this problem setting, we consider the following objective function:

$$\begin{aligned} \min_{\gamma_{X_S} \in \gamma_{X_S}} \quad & RNEL_{\gamma_{X_S}} \\ \text{s. t. } \quad & 0 < C_{\gamma_{X_S}} \leq c_0 \end{aligned} \tag{7}$$

where c_0 is the budget constraint.

Few studies have considered addressing a similar problem. Micheli et al. (2014) used the stochastic integer linear programming approach to select optimal strategies considering fuzzy-extended pairwise comparisons for the categories of risk impact. Aqlan and Lam (2015) used the Bow-Tie technique to identify and evaluate critical risks, and solved the multi-objective mixed-integer linear optimisation problem (objectives: total risk reduction, mitigation cost) using the goal programming. We consider a more complicated version of the problem where the $RNEL_{\gamma_{X_S}}$ value is calculated through running the BBN model for each combination of strategies. However, modelling the problem within the framework of BBNs makes it easier for the decision maker to only provide the effectiveness of each strategy in terms of reducing the probability and/or impact of related risk(s). Otherwise, it would be a daunting task to elicit these values from the decision maker in case of following the methods proposed by Aqlan and Lam (2015) and Micheli et al. (2014).

6. Demonstration of the Proposed Method

6.1 Description of the Case Study

We demonstrate the application of our proposed method through a simulation study. The study is based on the case study (Tuncel & Alpan, 2010) that was conducted in a medium-sized Turkish company involved in producing supplementary parts for electric, automotive and home appliance industries. Risk management is performed from the perspective of the manufacturer and only the immediate supply chain partners of the manufacturer are considered in the case study. Scope of the risk management is

confined to the four sub-systems of the supply chain: the inbound/outbound logistics; the operations at the manufacturer; the operations at the suppliers; and the final customers (via the retailer).

We make use of the same risks, associated risk sources and mitigation strategies in our simulation study that were identified in the case study through the FMEA. Mainly the existing causal dependency between individual risks and corresponding risk sources and strategies as reflected in the case study is maintained in our simulation study. However, in order to demonstrate the interdependency between different risk sources, risks and mitigation strategies, we have established arbitrary connections across seemingly possible causal factors. We used GeNIe (GeNIe, 2015) for modelling the network of risks and mitigation strategies. The qualitative structure of our model is shown in Fig. 7 whereas all the parameters used in the model are given in the Supplementary Material (see Appendix C). The oval shaped nodes indicate the uncertain variables representing both the risks and risk sources. Rectangular nodes represent different potential mitigation strategies and diamond shaped nodes represent the losses corresponding to different risks. It is important to realise that some mitigation strategies are directly connected to the risk sources or risks representing preventive strategies that reduce the probability of associated events. Risk mitigation strategies directly connected to the diamond shaped nodes represent reactive strategies that mitigate the impact of loss once the risk is realised.

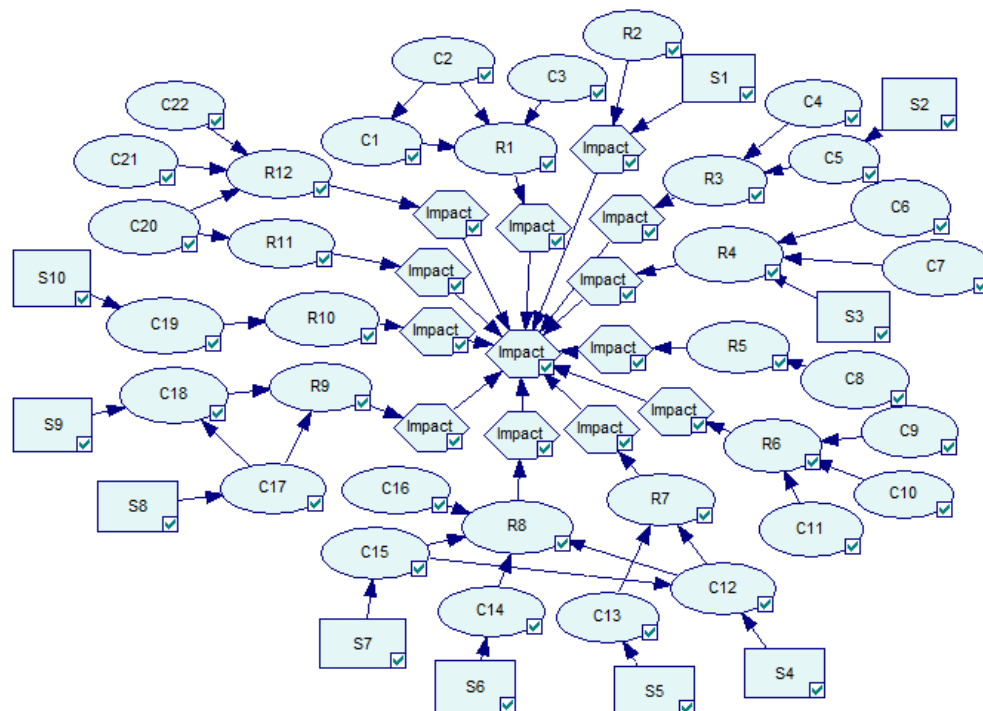


Fig. 7. Supply chain risks, risk sources and mitigation strategies modelled as a Bayesian Belief Network (GeNIe, 2015)

We have not used the ordinal data for the occurrence and severity for two reasons. Firstly, the occurrence data used in the FMEA does not consider the probabilistic interaction of risks and risk sources. Secondly, the use of ordinal data and subsequent multiplication of Occurrence, Severity and Detectability values for calculation of the RPN are mainly criticised in the literature for associated

shortcomings (Gilchrist, 1993; Nepal & Yadav, 2015). Therefore, we have assigned assumed probability values to all the uncertain nodes using the framework of BBNs. Although we have used the same values of severity appearing in the case study, we assume that these are the perceived loss values in the event of occurrence of relevant risks. Assumed costs associated with different mitigation strategies are shown in Table 6.

6.2 Results and Analysis

We focussed on two different scenarios. In the first scenario, we assumed that the strategies shown in Fig. 7 have not been already identified and the decision maker is interested in assessing risks first followed by mitigation of critical risks. Therefore, considering the decision maker as risk-neutral, we used the *RNELPM* to identify critical risks and subsequently used Shapley value to determine fair allocation of budget to mitigate the critical risks identified. In the second scenario, we considered the decision problem of optimising the strategies shown in Fig. 7 subject to different constraints. Here we assumed that the cost of strategies is already known and the strategies are fairly priced.

6.2.1 Scenario 1

We calculated the *RNELPM* values corresponding to all risks through propagating the impact of each risk across the risk network. In contrast with the conventional norm of mapping (independent) risks on a two-dimensional plane of probability and impact, we propose assessing the network wide exposure of each risk over the risk spectrum as shown in Fig. 8. The size of each bubble represents the product of probability and conditional expected loss related to each risk indicating its relative importance and rank. R7, R8 and R9 appear to be the most critical risks. Although R2 can pose a major threat to the network in case of its activation, its low probability does not necessitate mitigating the risk rather contingency plans may be tailored to deal with the risk.

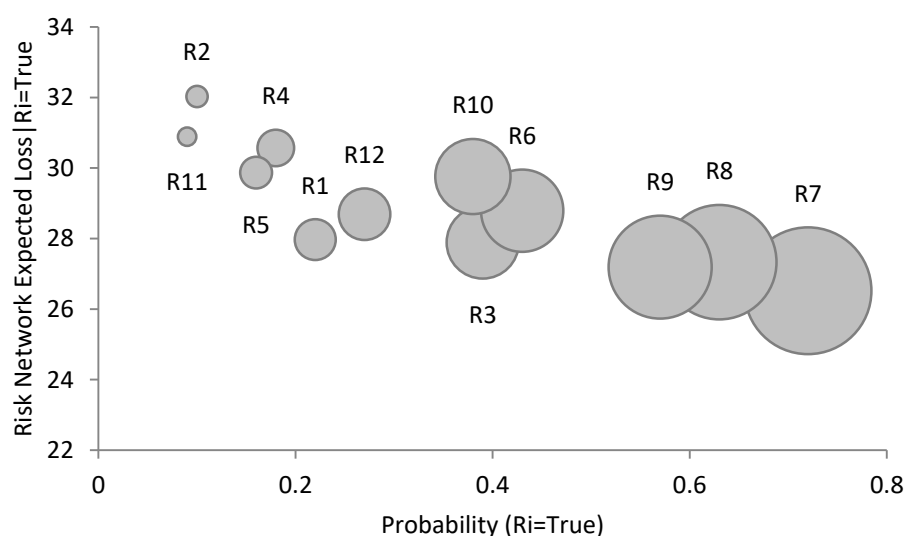


Fig. 8. Risk spectrum representing ranking of interdependent risks for the risk analysis stage with size of each bubble reflecting the relative value of RNELPM

Let us assume that the decision maker decides to mitigate the three critical risks identified. We determined the fair allocation of resources to deal with these risks using the Shapley value. The calculations are shown in Table 5. It can be seen that nearly equal budget should be allocated to the risks. However, it is important to realise that the allocation is a starting point as it might be possible to mitigate R7 at relatively lower cost. If these three risks are related to different suppliers, Shapley value helps in rewarding the suppliers fairly.

Table 5. Relative benefit of controlling each risk toward reduction in risk network expected loss

Control Risks $Z \cup \{i\}$	Expected Loss	Benefit of Control	Marginal Contribution $[v(Z \cup \{i\}) - v(Z)]$			Weight $\frac{ Z ! (N - Z - 1)!}{ N !}$
			$i = R7$	$i = R8$	$i = R9$	
0	24.59	0				
R7	20.22	4.37	4.37			1/3
R8	20.33	4.26		4.26		1/3
R9	21.19	3.4			3.4	1/3
R7, R8	17.21	7.38	3.12	3.01		1/6
R7, R9	16.82	7.77	4.37		3.4	1/6
R8, R9	16.92	7.67		4.27	3.41	1/6
R7, R8, R9	13.80	10.79	3.12	3.02	3.41	1/3
<i>Shapley value Φ_i</i>			3.75	3.64	3.4	
<i>Relative importance (%)</i>			34.76	33.73	31.51	

Once the critical risks are mitigated, there is a need for re-assessing the risks. Therefore, we re-calculated the *RNELPM* values for prioritising risks and developing contingency plans. In order to compare the values corresponding to the risk assessment and risk monitoring stages, we used the normalised *RNELPM* (with respect to $RNEL_{SC}$) as shown in Fig. 9. As R7, R8 and R9 have been completely mitigated, the normalised *RNELPM* value is shown as 0. R3, R6 and R10 need to be monitored owing to the higher measure values. The graph also helps in understanding the benefit of mitigating risks toward the risk network.

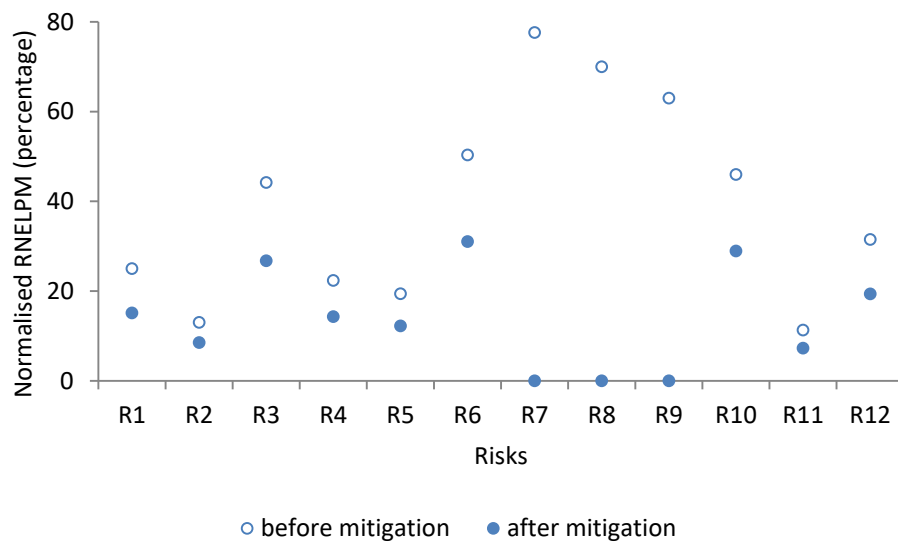


Fig. 9. Comparison of normalised RNELPM values corresponding to the risk analysis and risk monitoring stages

6.2.2 Scenario 2

Once the model was populated with all the parameters, it was updated in order to obtain an array of values (*RNEL*) corresponding to different combinations of mitigation strategies. We considered addressing two different problems of selecting optimal mitigation strategies under resource (number of strategies) and budget constraints (see section 5.2).

6.2.2.1 Prioritising Risk Mitigation Strategies under Resource Constraint

It is extremely important for the decision maker to select optimal cost-effective mitigation strategies under resource constraint as it might not be possible for the organisation to implement and manage all the strategies simultaneously. We consider the problem of selecting optimal strategies in relation to different objective functions (refer to Eqns (5) and (6)) and resource constraint (i.e. limited number of strategies can be applied). We updated the model in GeNIe (GeNIe, 2015) and exported the array of values to the Microsoft Excel worksheet in order to conduct the analysis. The results of optimal combination of strategies corresponding to the two objective functions are shown in Table 6. A decision maker might be faced with the problem of ranking mitigation strategies as in addition to the initial cost of implementing strategies, effort involved in managing the smooth execution of these strategies might be an important factor. The first scheme considers only the risk network expected loss without incorporating the cost element whereas the second scheme includes both the factors of improvement in risk network expected loss and associated cost of strategies.

Table 6. Prioritisation of optimal risk mitigation strategies corresponding to different objective functions and resource constraint

No. of Strategies	0	1	2	3	4	5	6	7	8	9	10
No. of Combinations	1	10	45	120	210	252	210	120	45	10	1
Optimal Strategies based on Minimum Risk Network Expected Loss		S10	S7, S10	S4, S7, S10	S4, S5, S7, S10	S4, S5, S7, S9, S10	All except S1, S2, S3 and S6	All except S1, S2 and S6	All except S2 and S6	All except S2	All
Risk Network Expected Loss	24.6	22.5	21.2	19.6	18.2	17.1	16.1	15.3	14.5	13.8	13.4
Improvement in Risk Network Expected Loss less Cost		0.1	0.4	1	1.4	1.4	0.5	-0.7	-1.9	-3.2	-3.8
Mitigation Cost	0	2	3	4	5	6	8	10	12	14	15
Optimal Strategies based on Maximum Improvement in Risk Network Expected Loss less Cost		S7	S4, S7	S4, S5, S7	S4, S5, S7, S9	S4, S5, S7, S9, S10	All except S1, S3, S6 and S8	All except S1, S3 and S6	All except S1 and S6	All except S6	All
Risk Network Expected Loss	24.6	23.3	21.6	20.3	19.2	17.1	16.7	15.7	14.9	14.1	13.4
Improvement in Risk Network Expected Loss less Cost		0.3	1.0	1.3	1.4	1.4	0.9	-0.1	-1.3	-2.5	-3.8
Mitigation Cost	0	1	2	3	4	6	7	9	11	13	15

Different combinations of mitigation strategies corresponding to the two objective functions and number of strategies are shown in Fig. 10 and Fig. 11. In both the graphs, it can be observed that there are a number of possible solutions to implementing specific number of strategies except the two options of implementing 'no strategy' and 'all strategies'. All combinations of strategies except the optimal combinations as mentioned in Table 6 are not optimal for managing risks.

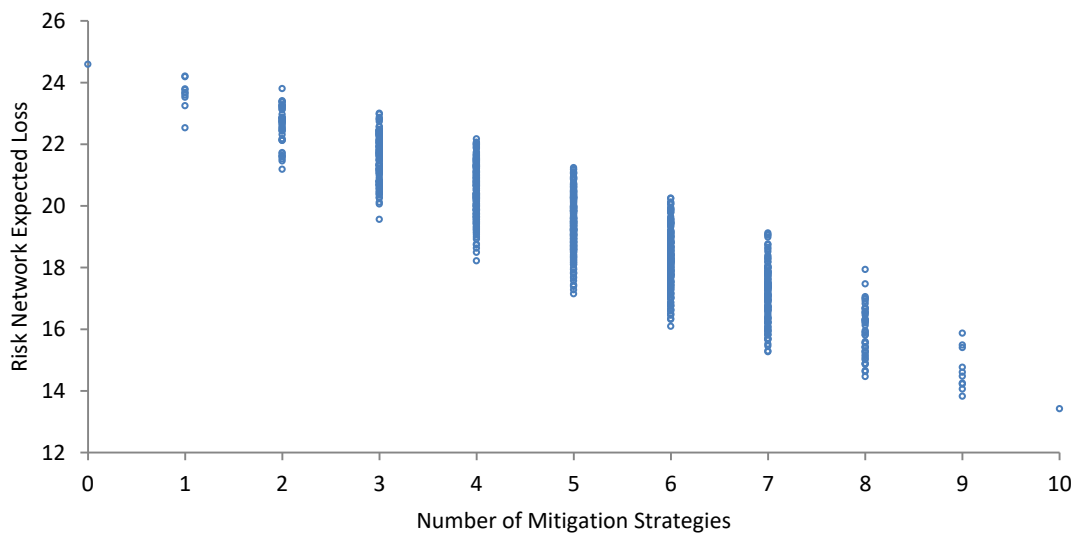


Fig. 10. Variation of risk network expected loss with the number of strategies

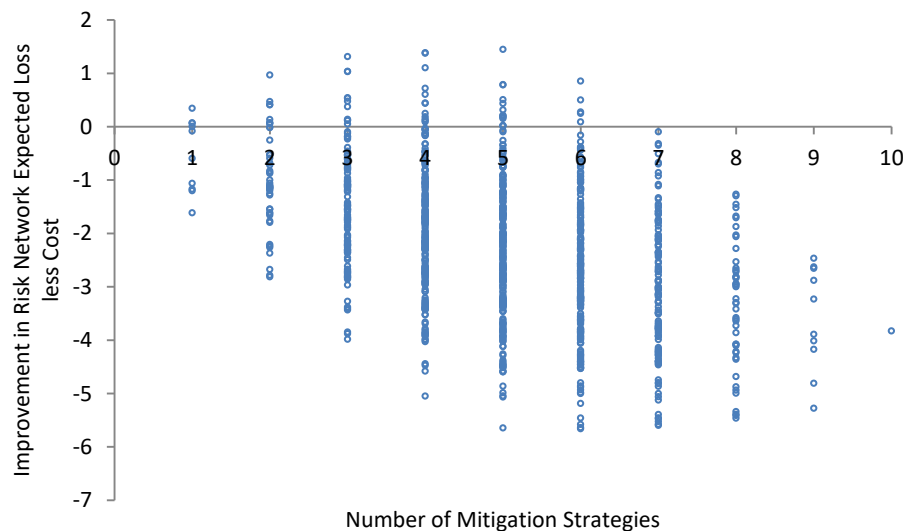


Fig. 11. Variation of improvement in risk network expected loss less cost with the number of strategies

6.2.2.2 Prioritising Risk Mitigation Strategies under Budget Constraint

In this problem setting, we consider the choice of selecting optimal strategies keeping in view the budget constraint. It can also be interpreted as a problem of selecting a cost-effective combination of mitigation strategies corresponding to a specific level of risk exposure (risk network expected loss). The results are shown in Table 7 which reveal the difference in selected combinations corresponding to the budget constraint. All combinations of strategies including the optimal solutions related to the objective

function are shown in Fig. 12. The optimal solutions for the objective function against specific budget constraint are represented by the corresponding lowest points. The graph indicates that the rate of improvement decreases with the increase in mitigation cost. Improvement in the risk network expected loss considering the cost of implementing strategies is shown in Fig. 13. Maximum net benefit (improvement in risk network expected loss less cost) is achieved at a cost of 6 units.

Table 7. Prioritisation of optimal risk mitigation strategies corresponding to the objective function with budget constraint

Mitigation Cost	Minimum Risk Network Expected Loss	
	Strategies	Risk Network Expected Loss
0	-	24.6
1	S7	23.3
2	S4, S7	21.6
3	S4, S5, S7	20.3
4	S4, S5, S7, S9	19.2
5	S4, S5, S7, S10	18.2
6	S4, S5, S7, S9, S10	17.1
7	All except S1, S3, S6 and S8	16.7
8	All except S1, S2, S3 and S6	16.1
9	All except S1, S3 and S6	15.7
10	All except S1, S2 and S6	15.3
11	All except S1 and S6	14.9
12	All except S2 and S6	14.5
13	All except S6	14.1
14	All except S2	13.8
15	All	13.4

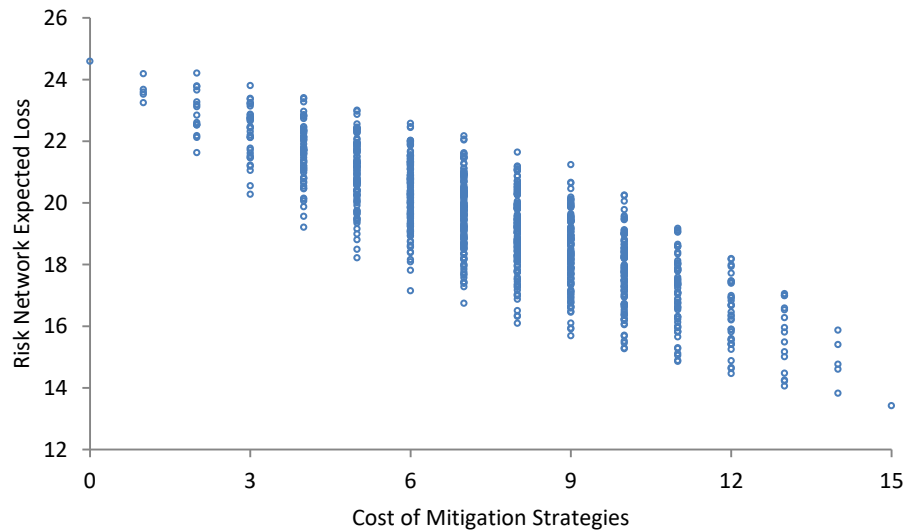


Fig. 12. Variation of risk network expected loss with the cost of strategies

Let us assume that the decision maker has implemented all potential strategies. In order to prioritise risks for the risk monitoring stage, we evaluated the *RNELPM* values for the risks as shown in Fig. 14. If we compare the results with the prioritisation results shown in Fig. 8, the conditional expected loss and the marginal probability values for all the risks are reduced substantially. R6 is the most significant risk for developing a contingency plan. Evaluation of risk mitigation strategies through our proposed approach helps in identifying an optimal mix of preventive and reactive strategies. As our approach

incorporates interdependency between supply chain risks, risk sources and mitigation strategies and follows a rigorous approach grounded in the theoretical framework of BBNs, the resulting solution can be considered as viable. However, it is assumed that the network structure and elicited values would truly reflect the real-time risk scenario. Adopting standard procedures of expert judgment can reduce the associated problems.

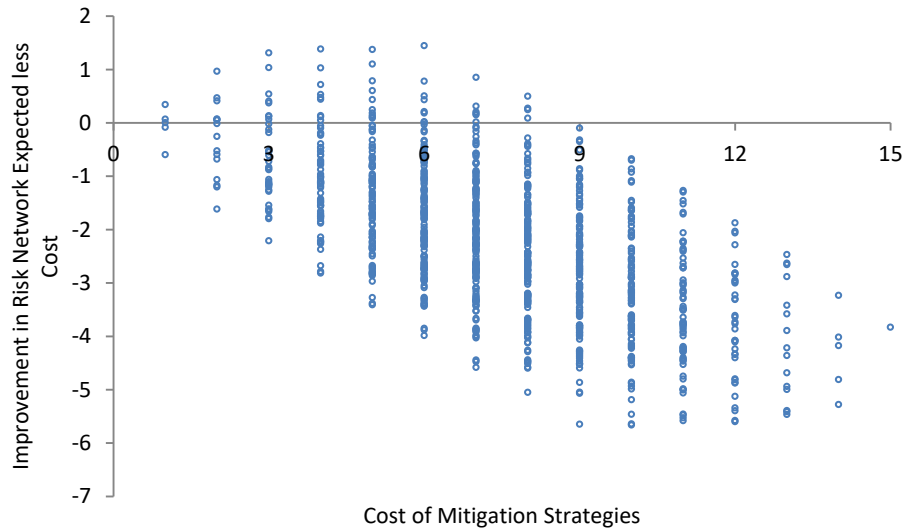


Fig. 13. Variation of improvement in risk network expected loss less cost with the cost of strategies

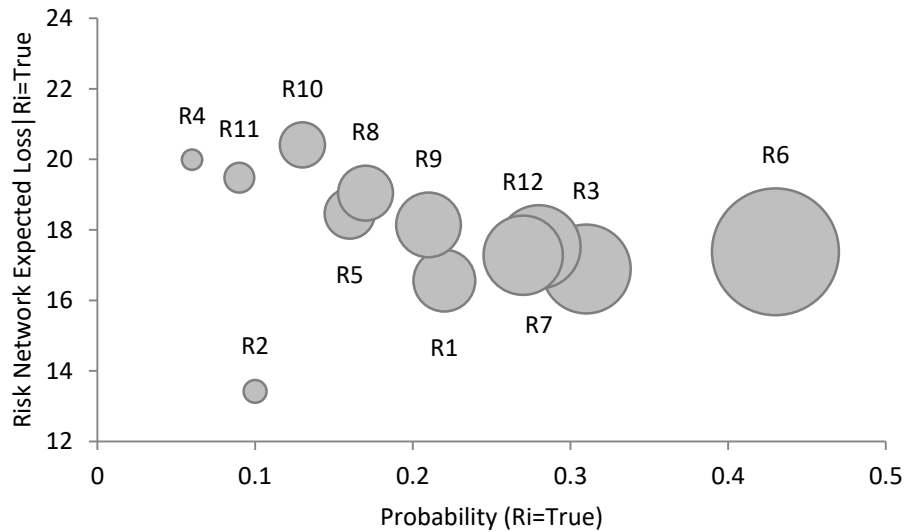


Fig. 14. Risk spectrum representing ranking of interdependent risks for the risk monitoring stage with size of each bubble reflecting the relative value of RNELPM

7. Conclusions

Current literature in SCRM has not considered the evaluation of risk mitigation strategies within a setting of interconnected risks and strategies involving the probabilistic interdependency between risks, losses resulting from the realisation of risks, and costs and relative benefits associated with different mitigation strategies. Moreover, existing risk measures do not capture the holistic network wide impact of risks and there is a need to develop dependency based measures that could be utilised in a specific

context. We proposed a SCRM process within the theoretically grounded framework of BBNs in order to bridge this major research gap. Modelling of risks through this approach is viable for even a huge network comprising many supply chain actors as opposed to the process mapping of a supply chain that involves brainstorming of risks following the supply network configuration. Our proposed method can help in determining an optimal mix of strategies in relation to budget and resource constraints.

We introduced dependency based risk measures for ranking risks and evaluating strategies that represent the relative contribution of each risk to the loss propagation across the network of interconnected risks in the scenario of its activation. Measures based on techniques other than BBNs are not able to capture the probabilistic interactions between risks and fail to account for causal and diagnostic inferencing. Using the concept of Shapley value, we have also devised a method to determine a fair allocation of resources to mitigate risks once the mitigation strategies with associated costs are not already established within a network setting.

There are few limitations of the study: we have only considered binary states for the risks and mitigation strategies; we have not modelled the detectability of risks as to how early a risk could be detected before its activation; and finally, the risk network captures a particular moment in time whereas the dynamic nature of the risk is not exclusively modelled. Future research may focus on representing risks by continuous variables and also, a control strategy may be represented by a continuum of control levels, associated effectiveness and costs. The proposed process may be applied in real case studies in order to evaluate its efficacy and be extended to account for strategic risks where the state of a risk is not driven by chance rather players within the supply network behave opportunistically and therefore, the actors make a choice based on maximising their expected utility value. Another important aspect is to model the detectability of risks as the response time before complete activation of a risk is a critical factor. Furthermore, the model can be extended to establish the source of defects within the supply chain especially in case of the food sector where it is hard to ascertain the main source of contamination.

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Appendix A: Proofs

Proof of Observation 1

$$RNEL = E[NL]$$

$$= E[NL|R_i = 0](1 - P(R_i = 1)) + E[NL|R_i = 1]P(R_i = 1)$$

$$= E[NL|R_i = 0] + P(R_i = 1)(E[NL|R_i = 1] - E[NL|R_i = 0])$$

As $E[NL|R_i = 1] > E[NL|R_i = 0]$ this is an increasing function in $P(R_i = 1)$ going from $E[NL|R_i = 0]$ to $E[NL|R_i = 1]$.

Proof of Observation 2

$$E_{NL}[\max(NL - E[NL], 0)] = E_{NL}[\max(NL - E[NL], 0)|R_i = 1]P(R_i = 1) + E_{NL}[\max(NL - E[NL], 0)|R_i = 0]P(R_i = 0)$$

From Observation 1 we know when $R_i = 1$ then $E[NL|R_i = 1] > E[NL]$ as well

$E[\max(NL, E[NL])|R_i = 1] \geq E[NL|R_i = 1]$ and therefore

$$\begin{aligned} E_{NL}[\max(NL - E[NL], 0)|R_i = 1] &\geq E[NL|R_i = 1] - E[NL] \\ &= \max(E[NL|R_i = 1] - E[NL], 0) \end{aligned}$$

From Observation 1 we know when $R_i = 0$ then $E[NL|R_i = 0] < E[NL]$ and therefore

$$\begin{aligned} E_{NL}[\max(NL - E[NL], 0)|R_i = 0] &\geq 0 \\ &= \max(E[NL|R_i = 0] - E[NL], 0) \end{aligned}$$

Proof of Observation 3

$$\frac{RNELPM_i - UTC_i}{P(R_i=1)} = \frac{E[NL|R_i=1]P(R_i=1) - (E[NL|R_i=1] - E[NL])P(R_i=1)}{P(R_i=1)}$$

$$= E[NL]$$

$$= RNEL$$

Proof of Proposition 1

From the proof of Observation 2 we have the inequality $E[\max(NL, E[NL])|R_i = 1] \geq E[NL|R_i = 1]$ which is an equality when $P(\text{Network Loss} < RNEL|R_i = 1) = 0$.

Proof of Proposition 2

$$\begin{aligned} UTC_i &= E_{R_i}[\max(E[NL|R_i = 1] - E[NL], 0)] \\ &= (E[NL|R_i = 1] - E[NL|R_i = 1]P(R_i = 1) - E[NL|R_i = 0]P(R_i = 0))P(R_i = 1) \\ &= (E[NL|R_i = 1] - E[NL|R_i = 0])P(R_i = 0)P(R_i = 1) \\ &= (E[NL|R_i = 1]P(R_i = 1) - E[NL|R_i = 0]P(R_i = 1))P(R_i = 0) \end{aligned}$$

$$\begin{aligned}
&= (E[NL|R_i = 1]P(R_i = 1) - E[NL|R_i = 0] + E[NL|R_i = 0]P(R_i = 0))P(R_i = 0) \\
&= (E[NL] - E[NL|R_i = 0])P(R_i = 0) \\
&= LTG_i
\end{aligned}$$

Proof of Proposition 3

$$\begin{aligned}
UTC_i &= E_{R_i}[\max(E[NL|R_i = 1] - E[NL], 0)] \\
&= P(R_i = 1)(E[NL|R_i = 1] - E[NL|R_i = 1]P(R_i = 1) - E[NL|R_i = 0]P(R_i = 0)) \\
&= P(R_i = 0)P(R_i = 1)(E[NL|R_i = 1] - E[NL|R_i = 0]) \\
&= \sigma_i^2[E[NL|R_i = 1] - E[NL|R_i = 0]]
\end{aligned}$$

Proof of Corollary 1

As $UTC_i = \sigma_i^2[E[NL|R_i = 1] - E[NL|R_i = 0]] = P(R_i = 0)P(R_i = 1)(E[NL|R_i = 1] - E[NL|R_i = 0])$ this can be reduced to 0 either by setting $P(R_i = 0) = 0$ or $P(R_i = 1) = 0$. If we assume that costs increase with reducing $P(R_i = 1)$ then the optimal target for each risk would be $P(R_i = 1) = 0$.

Proof of Proposition 4

The optimisation problem is

$$\min \sum_i RNELPM_i \text{ s.t. } \sum_i C_i(P_i) = c_0$$

The Lagrangian function for this constrained optimisation problem is

$$O = \sum_i P_i E[L|R_i = 1] + \lambda(\sum_i C_i(P_i) - c_0)$$

And the first-order conditions (FOC) are

$$O_i = E[L|R_i = 1] + \lambda C'_i(P_i) = 0 \forall i$$

and

$$O_\lambda = \sum_i C_i(P_i) - c_0 = 0$$

This gives us

$$1/\lambda \begin{bmatrix} E[L|R_1 = 1] \\ \vdots \\ E[L|R_n = 1] \end{bmatrix} = \begin{bmatrix} -C'_1(P_1) \\ \vdots \\ -C'_n(P_n) \end{bmatrix} \text{ s.t. } \sum_i C_i(P_i) = c_0$$

Proof of Lemma 1

From FOC we have

$$\frac{1}{\lambda^*} \begin{bmatrix} E[L|R_1 = 1] \\ \vdots \\ E[L|R_n = 1] \end{bmatrix} = \begin{bmatrix} -C'_1(P_1) \\ \vdots \\ -C'_n(P_n) \end{bmatrix} \text{ s.t. } \sum_i C_i(P_i) = c_0$$

The unconstrained optimal solution has $\lambda^* = 1$. Assuming $\text{sign}(C''_i(P_i)) = \text{sign}(C''_j(P_j)) \forall i, j$ decreasing λ^* will result in higher costs (i.e. a lower P_i) only if $\text{sign}(C''_i(P_i)) > 0$ and increasing λ^* will result in higher costs only if $\text{sign}(C''_i(P_i)) < 0$.

Proof of Proposition 5

We now seek to understand how the optimal risk mitigation strategy changes with a relaxation in the budget. Writing $P_i^*(c_0)$ for the optimal solution for each i , which is implicitly defined by the relationship $-C'_i(P_i^*(c_0)) = \left(\frac{1}{\lambda^*}\right) E[L|R_i = 1]$, we differentiate each of the first-order conditions with respect to c_0 to deduce

$$\begin{aligned} \lambda C''_i \frac{dP_i^*(c_0)}{dc_0} + C'_i \frac{d\lambda}{dc_0} &= 0 \quad \forall i \\ C'_i \frac{dP_i^*(c_0)}{dc_0} + \sum_{j \neq i} C'_j \frac{dP_j^*(c_0)}{dc_0} - 1 &= 0 \end{aligned}$$

The i th equation in the first set gives

$$\frac{d\lambda}{dc_0} = -\lambda \frac{C''_i}{C'_i} \frac{dP_i^*(c_0)}{dc_0}$$

Each of the other equations in the first set gives for each $j \neq i$

$$\begin{aligned} \frac{dP_j^*(c_0)}{dc_0} &= -\frac{d\lambda}{dc_0} \frac{1}{\lambda} \frac{C'_j}{C''_j} \\ &= \frac{C''_i}{C'_i} \frac{dP_i^*(c_0)}{dc_0} \frac{C'_j}{C''_j} \end{aligned}$$

(using $\frac{d\lambda}{dc_0}$ just deduced). Inserting these objects into the final equation gives

$$\begin{aligned} C'_i \frac{dP_i^*(c_0)}{dc_0} + \frac{C''_i}{C'_i} \frac{dP_i^*(c_0)}{dc_0} \sum_{j \neq i} \frac{C_j'^2}{C_j''} &= 1 \\ \frac{dP_i^*(c_0)}{dc_0} &= \frac{1}{C'_i + \frac{C''_i}{C'_i} \sum_{j \neq i} \frac{C_j'^2}{C_j''}} \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{C'_i + \sum_{j \neq i} C'_j \frac{A_i}{A_j}} \\
&= \frac{1}{C'_i \frac{A_i}{A_i} + \sum_{j \neq i} C'_j \frac{A_i}{A_j}} \\
&= \frac{1}{\sum_{k=1}^n C'_k \frac{A_i}{A_k}}
\end{aligned}$$

where $A_i = \frac{C''_i}{C'_i}$ is the Arrow Pratt measure of risk aversion applied to the cost function that measures its curvature, and n is the total number of risks within the network.

This allows us to understand how the probability of a particular risk would change through mitigation if extra budget became available. When seeking to allocate additional resource to mitigating risk a decision maker will be seeking to reduce probabilities in the most effective way, and this depends on the curvature of the cost function: a given additional resource to a risk will reduce the probability of it occurring more if the cost function is less convex. As such, when the ratio C''_i/C'_i is small in absolute terms for a risk, $\frac{dP_i^*(c_0)}{dc_0}$ will be larger in absolute terms (i.e. $P_i^*(c_0)$ is steeper) meaning the risk will be mitigated more when additional resource is available.

Appendix B: Flow Charts

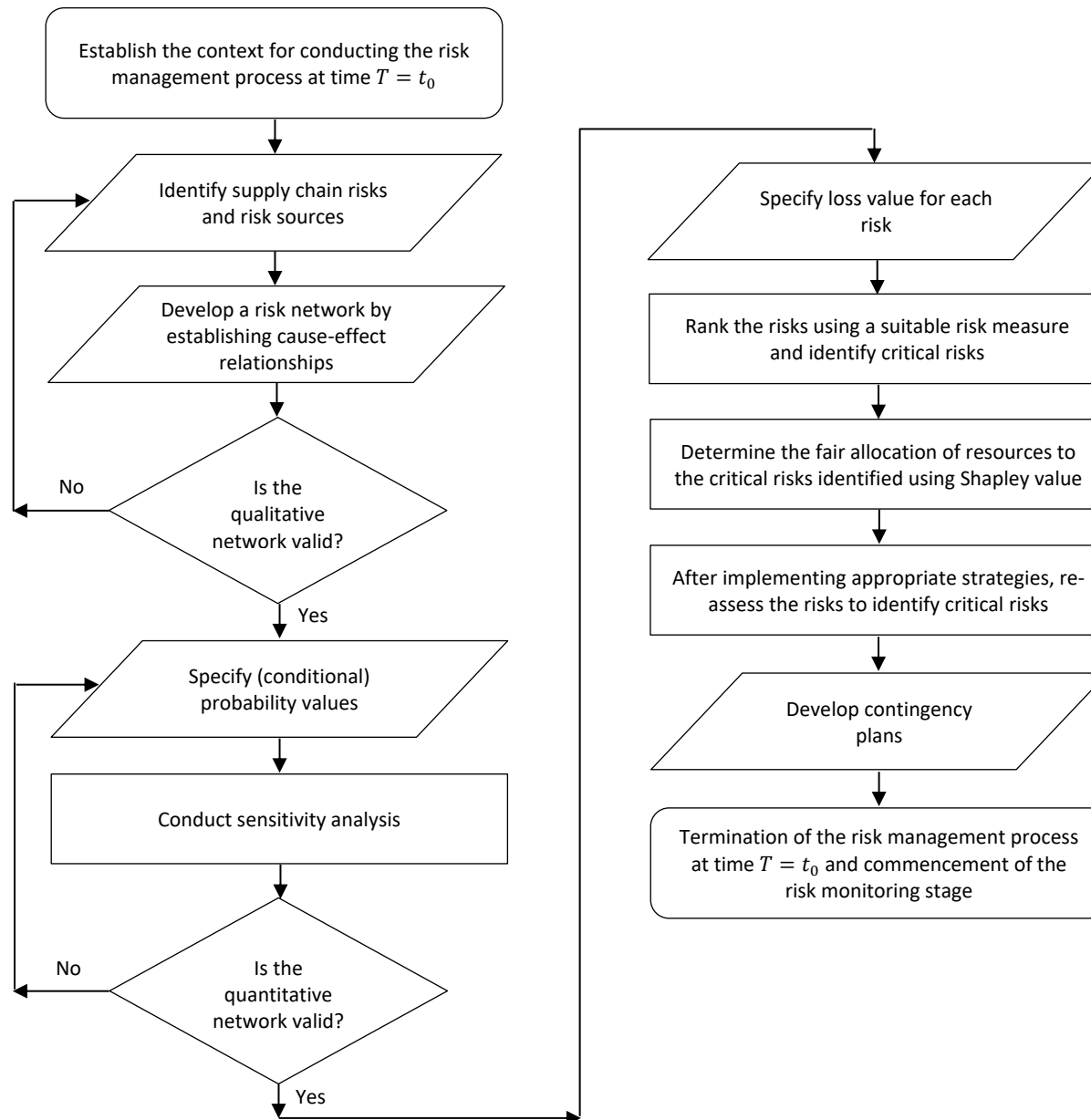


Fig. B.1. Flow chart for implementing the process where the strategies and associated cost are not already established

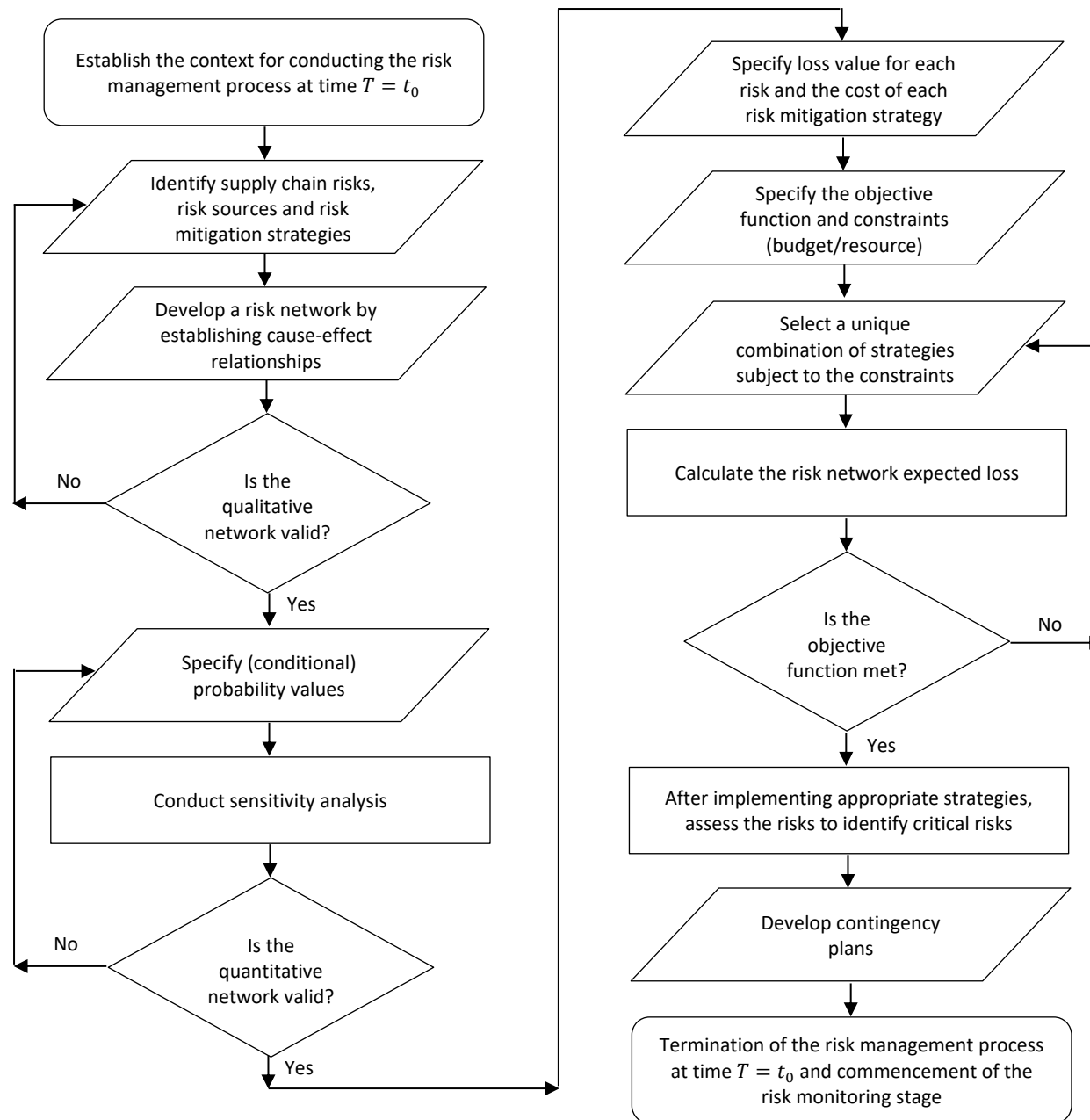


Fig. B.2. Flow chart for implementing the process where the strategies and associated cost are already established

Appendix C: Model Parameters

Table C.1. Description of risk sources, risks and mitigation strategies [adapted from Tuncel and Alpan (2010)]

Symbol	Risk Source (C)/Risk (R) [associated supply chain domain]/Strategy (S)
C1	Stress on crew
C2	Long working times
C3	Lack of training
C4	Negligence in maintenance
C5	Old technology
C6	High competition
C7	Opportunistic behaviour
C8	Decline in Customer Resource Management function
C9	Instable manufacturing process
C10	Low technical reliability
C11	Insufficient maintenance
C12	Dissatisfaction with work
C13	Strikes
C14	Lack of training
C15	Poor working conditions
C16	Insufficient breaks
C17	Planning and scheduling errors
C18	Bullwhip effect
C19	Low technical reliability
C20	Technological changes
C21	Contractual problems
C22	Monopoly
R1	Human error [Inbound/Outbound Logistics]
R2	Natural hazards [Inbound/Outbound Logistics]
R3	Technical problems with transportation vehicles [Inbound/Outbound Logistics]
R4	Loss of market share [Customers]
R5	Fluctuations in customer demands [Customers]
R6	Technical problems [Manufacturer]
R7	Absence of operator [Manufacturer]
R8	Human error [Manufacturer]
R9	Scarcity of raw parts [Suppliers]
R10	Poor quality in purchased products from supplier [Suppliers]
R11	Loosing competitive advantage of supplier [Suppliers]
R12	Decline in business relations with supplier [Suppliers]
S1	Insurance
S2	Capital investment
S3	R&D and marketing strategies
S4	Reward system
S5	Good relations with labour union
S6	Training
S7	Ergonomic Awareness program
S8	Investment in Enterprise Resource Planning
S9	Information sharing with supplier
S10	Rigorous process of Supplier selection

Table C.2. Probability values of root nodes

Causes or Risks (X_i)	$P(X_i = True)$
C2	0.2
C3	0.1
R2	0.1
C4	0.3
C6	0.05
C7	0.1
C8	0.2
C9	0.3
C10	0.4
C11	0.2
C16	0.1
C20	0.1
C21	0.3
C22	0.1

Table C.3. Conditional probability values of child nodes

<i>Parents</i>										$P(X_i = True Parents)$					
C1	C2	C3	C4	C5	C6	C7	C8	S2	S3	C1	C5	R1	R3	R4	R5
	T									0.7					
	F									0.3					
T	T	T										0.9			
T	T	F										0.5			
T	F	T										0.7			
T	F	F										0.3			
F	F	T										0.6			
F	T	F										0.3			
F	T	T										0.7			
F	F	F										0.01			
								Y			0.1				
								N			0.4				
			T	T									0.9		
			T	F									0.7		
			F	T									0.4		
			F	F									0.1		
					T	T				Y				0.4	
					T	F				Y				0.3	
					F	T				Y				0.3	
					F	F				Y				0.02	
					T	T				N				0.98	
					T	F				N				0.5	
					F	T				N				0.7	
					F	F				N				0.1	
							T								0.4
							F								0.1

Table C.3. Conditional probability values of child nodes (continued)

[illegible]

Table C.3. Conditional probability values of child nodes (continued)

<i>Parents</i>												$P(X_i = \text{True} \text{Parents})$						
C9	C10	C11	C12	C13	C14	C15	C16	S4	S5	S6	S7	R6	C12	C13	C14	C15	R7	R8
T	T	T										0.99						
T	F	T										0.8						
T	T	F										0.8						
T	F	F										0.6						
F	T	F										0.6						
F	F	T										0.5						
F	T	T										0.8						
F	F	F										0.01						
						T		Y					0.7					
						F		Y					0.02					
						T		N					0.8					
						F		N					0.4					
									Y					0.2				
									N					0.6				
										Y					0.2			
										N					0.5			
											Y					0.1		
											N					0.6		
			T	T													0.99	
			T	F													0.6	
			F	T													0.8	
			F	F													0.1	
			T		T	T	T											0.99
			T		T	T	F											0.9
			T		T	F	T											0.9
			T		T	F	F											0.8
			T		F	T	T											0.8
			T		F	T	F											0.75
			T		F	F	T											0.7
			T		F	F	F											0.65
			F		T	T	T											0.7
			F		T	T	F											0.6
			F		T	F	T											0.5
			F		T	F	F											0.4
			F		F	T	T											0.5
			F		F	T	F											0.4
			F		F	F	T											0.2
			F		F	F	F											0.01

Table C.4. Loss values of risks [adapted from Tuncel and Alpan (2010)]

Symbol	Risk	Loss
R1	Human error	4
R2	Natural hazards	8
R3	Technical problems with transportation vehicles	5
R4	Loss of market share	7
R5	Fluctuations in customer demands	6
R6	Technical problems	7
R7	Absence of operator	5
R8	Human error	6
R9	Scarcity of raw parts	6
R10	Poor quality in purchased products from supplier	8
R11	Loosing competitive advantage of supplier	6
R12	Decline in business relations with supplier	5

Table C.5. Costs associated with mitigation strategies

Symbol	Mitigation Strategy	Cost
S1	Insurance	2
S2	Capital investment	1
S3	R&D and marketing strategies	2
S4	Reward system	1
S5	Good relations with labour union	1
S6	Training	2
S7	Ergonomic Awareness program	1
S8	Investment in Enterprise Resource Planning	2
S9	Information sharing with supplier	1
S10	Rigorous process of Supplier selection	2

References

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